### Semester Two Examination, 2017

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**SPECIALIST**

**UNITS 1 AND 2**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student Number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working  time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 48 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 95 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (95 Marks)

This section has**thirteen (****13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

An exam has two parts, and , containing 15 and 8 questions respectively.

Determine the number of different combinations of questions a candidate could choose if they must answer

(a) 5 questions from part and 4 questions from part . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses multiplication of combinations  ✓ correct number |

(b) 3 questions, all chosen from the same part. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses addition of combinations  ✓ correct number |

(c) 3 questions, with at least one question from each part. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates total ways, no restriction  ✓ subtracts answer from (b) |

Question 10 (6 marks)

(a) The point is translated by the column vectors and to . Determine the values of the constants and . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of  ✓ value of |

(b) Determine the single matrix that represents, in order, the composition of a reflection in the line followed by an anti-clockwise rotation of about the origin. Express matrix coefficients in exact form. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ matrix for rotation  ✓ matrix for reflection  ✓ multiplies in correct order  ✓ correct matrix |

Question 11 (6 marks)

(a) A circle property says that if chords of a circle are of equal length then they subtend equal angles at the centre.

(i) Write the converse of this statement. (1 mark)

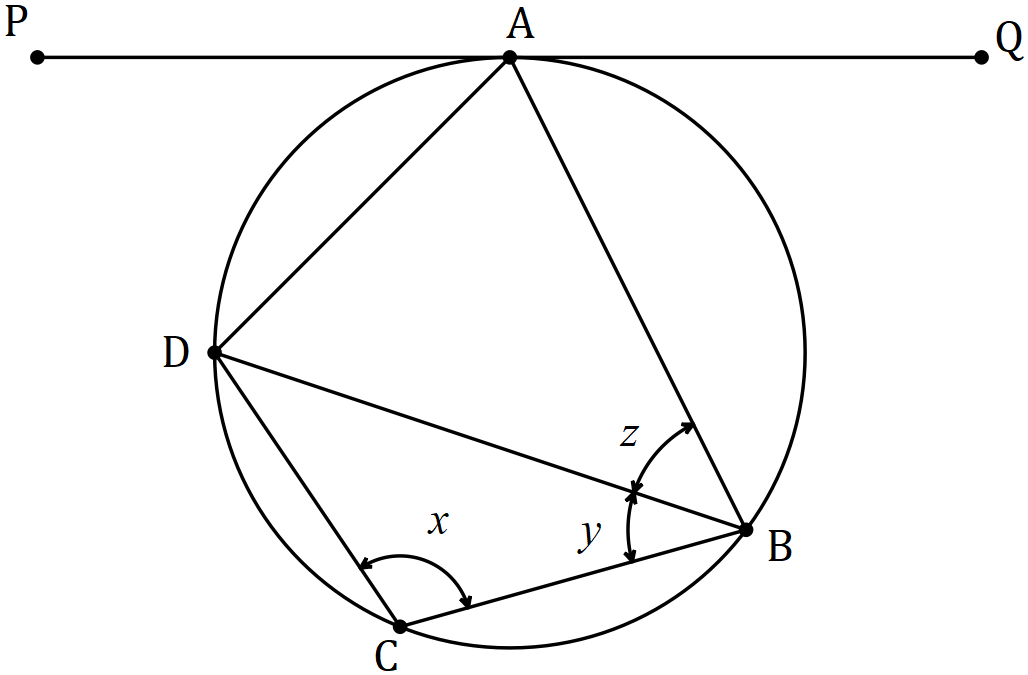
|  |
| --- |
| **Solution** |
| If chords of a circle subtend equal angles at the centre then they are of equal length. |
| **Specific behaviours** |
| ✓ writes converse |

(ii) Draw a diagram to illustrate the converse statement and state whether the converse is also true. (2 marks)

|  |
| --- |
| **Solution** |
| Converse is true. |
| **Specific behaviours** |
| ✓ diagram  ✓ states converse true |

(b) The diagram below shows four points , , and lying on the circumference of a circle. The line is a tangent to the circle at , , and .

Determine the size of angles , and . (3 marks)



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓✓✓ each angle |

Question 12 (9 marks)

(a) If and determine

(i) the angle between the directions of and , to the nearest tenth of a degree.

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states angle  ✓ rounds correctly |

(ii) the scalar projection of on . (2 marks)

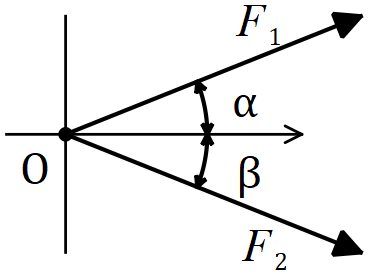
|  |
| --- |
| **Solution** |
| Exact: |
| **Specific behaviours** |
| ✓ expression for projection  ✓ states value |

(b) The vector has a magnitude of and is perpendicular to the vector . Determine the values of the constants a and b, where . (5 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses magnitude to form equation  ✓ calculate values of a  ✓ uses dot product to form equation  ✓ calculate values of b  ✓ chooses correct pairing |

Question 13 (8 marks)

In the diagram below, forces and act on a body at the origin.



(a) If , , and , determine the magnitude of the resultant force and the angle it makes with the positive axis. (5 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ sketch with forces nose to tail  ✓ uses cosine rule for magnitude  ✓ states magnitude  ✓ uses sine rule for angle  ✓ states angle with axis |

(b) If and , determine the angles and so that the resultant force is directed along the positive axis and has a magnitude of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ sketch with resultant on axis  ✓ uses cosine rule for angle  ✓ uses sine or cosine rule for second angle |

Question 14 (6 marks)

(a) Prove that . (4 marks)

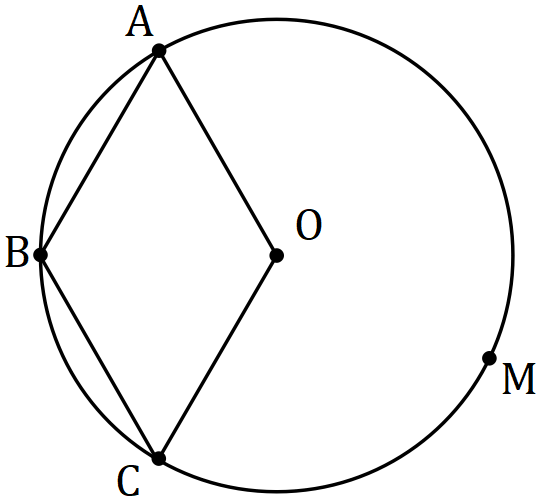
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expands sum of A and 2A  ✓ uses double angle identity for  ✓ eliminates  ✓ simplifies |

(b) Hence, or otherwise, solve . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes as  ✓ solution |

Question 15 (7 marks)

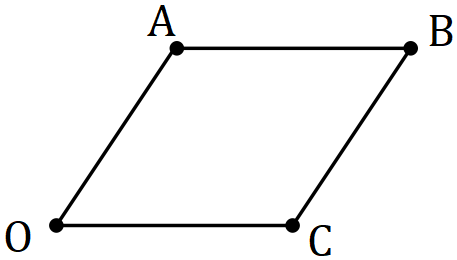
(a) The diagram below shows vertices , and of rhombus lying on the circumference of circle centre and point lying on the major arc . Determine the size of angle . (3 marks)



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates triangle equilateral  ✓ angle  ✓ angle |

(b) Use a vector method to prove that the diagonals of a rhombus are perpendicular.

(4 marks)

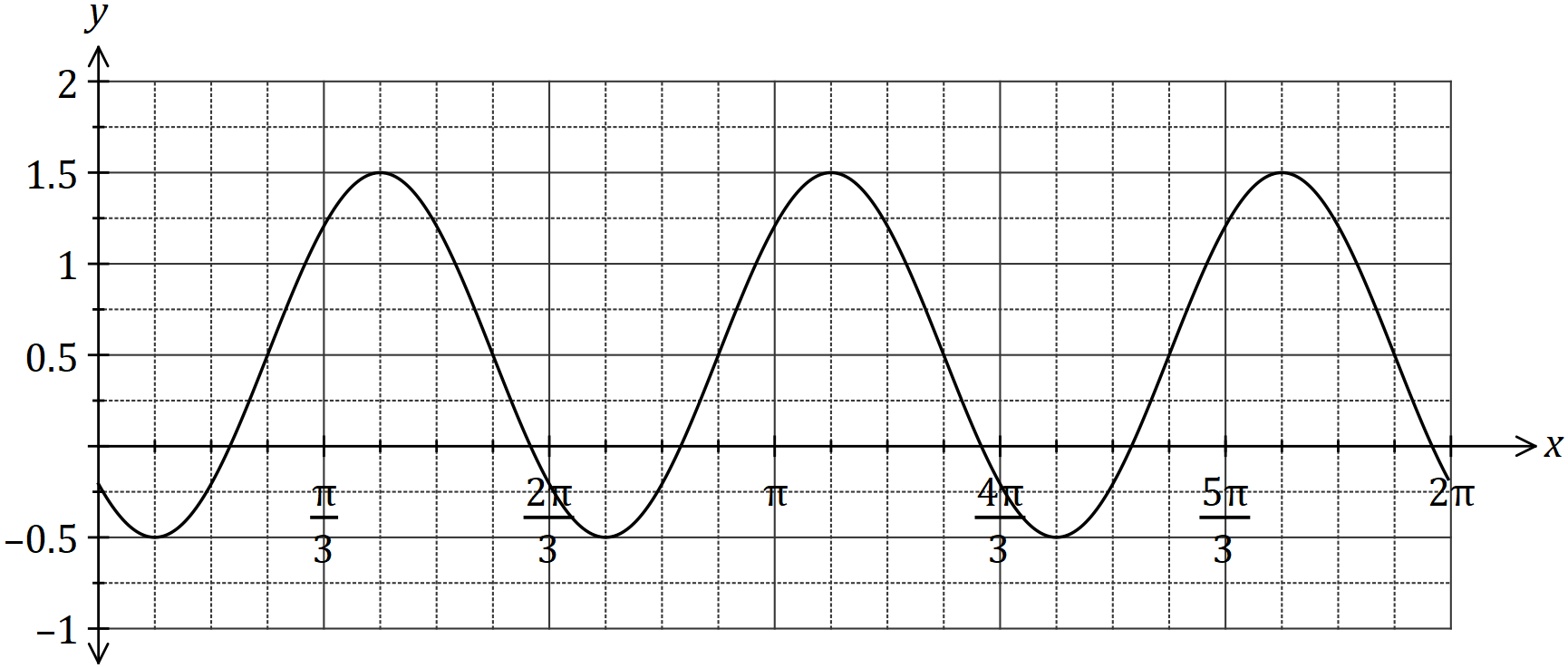


Let and .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ vectors for ,  ✓ forms scalar product  ✓ simplifies scalar product, with reasons  ✓ concludes perpendicular |

Question 16 (10 marks)

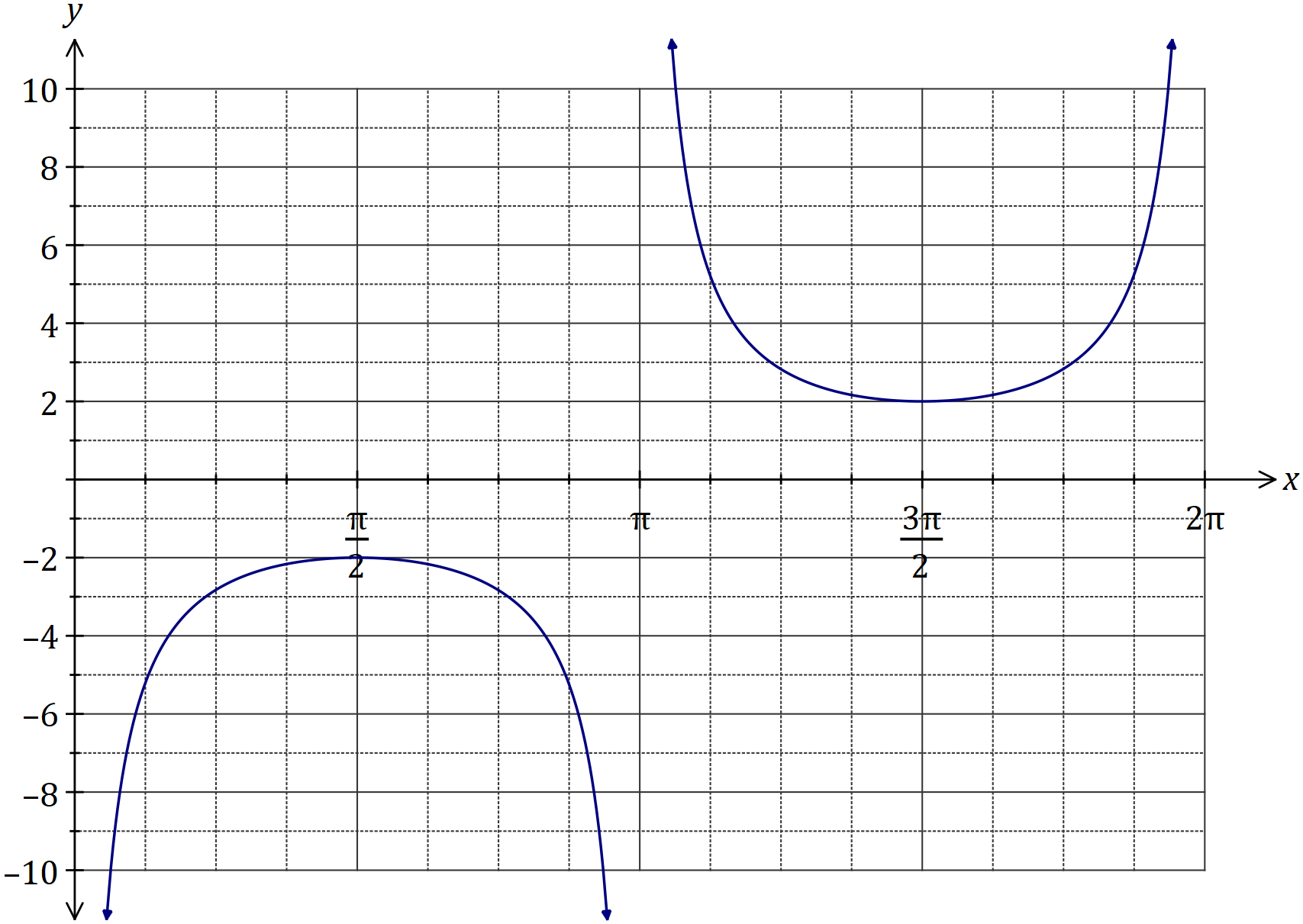
(a) The graph of is shown below for .



Determine the value of the positive constants , and . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of , ✓ value of b, ✓ value of |

(b) On the axes below, sketch the graph of , . (3 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ asymptotic behaviour  ✓ locations of max/min  ✓ smooth curve |

(c) The displacement, cm, of a particle from a fixed point varies with time, seconds, according to the model . Determine

(i) the initial displacement of the particle from . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(ii) the exact amplitude of the motion. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(iii) the period of motion. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(iv) the first time that the particle passes through , rounded to two decimal places.

(1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

Question 17 (4 marks)

A number is formed using five different digits chosen from those in the number 681 429. Determine how many different numbers can be formed that are

(i) odd. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states number |

(ii) greater than 90 000. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states number |

(iii) odd or greater than 90 000. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates number odd and greater than 90 000  ✓ states number |

Question 18 (9 marks)

Triangle has vertices , and .

(a) The vertices are transformed to using matrix . Write down the new coordinates of the vertices and describe the transformation. (4 marks)

|  |
| --- |
| **Solution** |
| Transformation is a reflection in the line . |
| **Specific behaviours** |
| ✓ matrix product  ✓ writes as coordinates  ✓ states reflection  ✓ states equation of line of reflection |

(b) The vertices are transformed to by matrix so that the new coordinates of the vertices are , and .

(i) Determine the transformation matrix . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes matrix equation  ✓ uses inverse  ✓ determines |

(ii) If the area of triangle is square units, express the area of triangle in terms of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determinant of  ✓ expresses area |

Question 19 (10 marks)

(a) Trapezium has parallel sides and . is the midpoint of and lies on so that .

Given that , and , determine the following in terms of and .

(i) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates half of  ✓ correct vector |

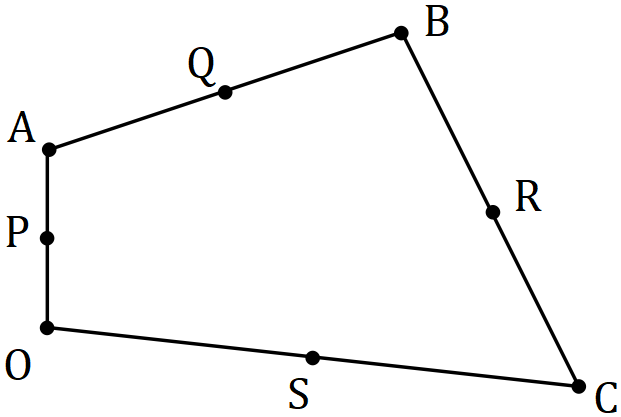
(ii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates and third of  ✓ correct vector |

(iii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates difference of (i) and (ii)  ✓ correct vector |

(b) Quadrilateral is shown below, where , , and are the midpoints of the sides , , and respectively. Let , and .



Show that . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ vector  ✓ vector  ✓ vector  ✓ vector |

Question 20 (8 marks)

The sum of the first terms of the sequence is .

(a) Show that this statement is true when . (2 marks)

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| --- |
| **Solution** |
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| **Specific behaviours** |
| ✓ shows sum of terms for LHS  ✓ shows substitution in RHS and states true |

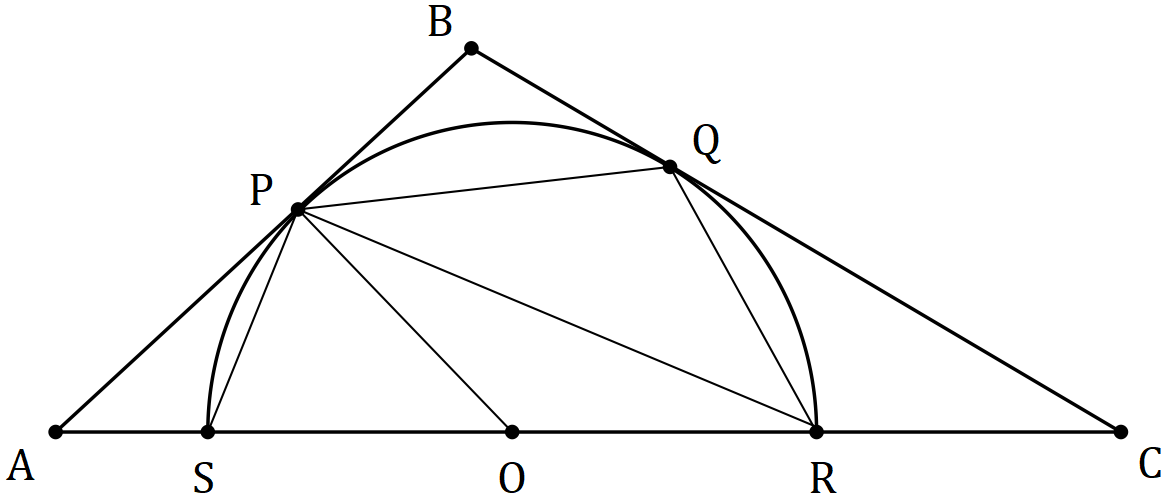
(b) Use mathematical induction to prove the statement is true for . (6 marks)

|  |
| --- |
| **Solution** |
| :  : |
| **Specific behaviours** |
| ✓ assumed true for  ✓ adds next term to both sides  ✓ simplifies RHS  ✓ factors k out of RHS  ✓ indicates true for  ✓ summary statement including truth of |

Question 21 (6 marks)

The diagram shows a semi-circle, with diameter and centre , circumscribed by triangle , in which and .

Determine, with reasons, the size of angles and .



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓  ✓  ✓ reasoning  ✓  ✓  ✓ reasoning |

Additional working space

Question number: \_\_\_\_\_\_\_\_\_

Additional working space

Question number: \_\_\_\_\_\_\_\_\_